School of Mathematics and Statistics

Faculty of Science, Technology, Engineering and Mathematics

2025 PhD Projects

Project title	Lacunarity, Anisotropy and Shape Statistics of Dynamical Fractals
Principal supervisor	Michael Wilkinson
Second supervisor	Marc Pradas or Katrine Rogers Applied mathematics
Discipline	Applied mathematics
Research area/keywords	Fractals, Strange attractors, Scattering
Suitable for	Either full time or part time applicants

Project background and description

Fractal sets [1] appear naturally in models for many physical processes: examples include the distributions of galaxies in the cosmos, the distributions of stars within galaxies [2], the distribution of debris floating on the surface of the ocean [3], and the distribution of small particles, such as water droplets in clouds, in a turbulent flow. The underlying models are typically dynamical systems which have chaotic motion for which the measure converges to a strange attractor [4]. The fractal structure is usually characterised by considering the mass inside a small ball, which satisfies a power law relation with an exponent which is known as the fractal dimension [1]. This project is concerned with developing novel approaches to characterising the structure of physically important fractal sets, going beyond defining fractal dimensions. One aspect concerns the distribution of matter in regions of very low density. There is recent evidence that regions of very low density have a power-law distribution of density, which is distinct from the power-law distribution of high densities which defines the fractal dimension. Part of the project will investigate a 'lacunarity exponent', which characterises the sparsely occupied regions. Another aspect is that recent work has shown that the distribution of matter can can become increasingly anisotropic when examined on finer scales [5]. The project will also address how the statistics of the shape of 'constellations' of points in the fractal may become increasingly anisotropic when examined on an increasingly fine scale [6]. The physical implications for scattering of light from fractal distributions of matter will also be investigated, and possible applications in remote sensing and astrophysical imaging will be assessed.

Background reading/references

1 K. Falconer, *Fractal Geometry: mathematical foundations and applications*, Wiley, New York, (1990).

2 B. J. Elmegreen and D. M. Elmegreen, *Fractal structure in galactic star fields*, *Astrophys. J.*, **121**, 1507-11, (2001).

3 J. Larkin, M. M. Bandi, A. Pumir and W. I. Goldburg, *Power-law distributions of particle concentration in free-surface flows*, *Phys. Rev. E*, **80**, 066301,(2009).

4 E. Ott, *Chaos in Dynamical Systems*, 2nd edition, Cambridge: University Press, (2002).

5 M. Wilkinson, H. R. Kennard and M. A. Morgan, *Spectal Dimension of Fractal Sets*, *J. Phys. A: Math. Theor.*, **45**, 415102, (2012).

6 M. Wilkinson and J. Grant, *Triangular Constellations in Fractal Measures*, *Europhys. Lett.*, **107**, 50006, (2014).